ASPECTS REGARDING REDUCTION OF SLIDING VECTORS

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Abstract: The paper presents certain aspects regarding reduction of sliding vectors, namely the case of a system made up of two vectors having any straight supports in space.

Keywords: reduction, sliding vectors

1. INTRODUCTION

As it is known, a vector is a physical quantity defined by three properties: numeric value, direction and sense. The direction of a vector is a line in the space of existence of a vector that is parallel with the vector. The line that is colinear with the vector, that is, the line obtained by prolonging the vector beyond the origin and extremity of the vector, is called support line (axis of the vector).

The versor of a vector axis is the vector the length of which is equal to the measurement unit of the respective vector. The versor specifies a positive sense on the vector axis. If the vector has the same sense with the versor, its algebraic value is positive, if the sense is opposite to the versor, its value is negative.

By vector module, the module of its algebraic value is understood, a positive number equal to the length of the vector related to the measurement unit.

Substituting the concept of numeric value with the module and concept of direction with the support line, one can say that a vector is defined by three properties: module, support line and sense.

In classical mechanics, according to the origin type, the following vector categories (classes) are defined: free vectors, the support line of which can take any position in space, parallel with the given direction, the application point (origin) not

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being specified; sliding vectors, the support line of which is stable in space and the application point (vector origin) is free on the support line; connected vectors, the support line of which is stable in space and the application point is fixed on the support line.

2. SLIDING VECTORS

Sliding vector model corresponds to the action of forces on solid rigid bodies (that cannot go out of shape). A force acting on a solid rigid body produces the same effect, irrespective of the point of force application on the support line, considered fixed in relation to the body. Therefore, the forces acting on a solid rigid body present themselves as sliding vectors and fall into this category of vectors.

Several forces acting on a solid rigid body form a system of forces.

If two different force systems, successively acting on a free solid rigid body, always found in the same state of movement, identically modifies the state of motion of the body, then the two systems are equivalent from the point of view of mechanical movement. If two different force systems, successively acting on a solid rigid body immobilized by bonds, determine the same reactions in the bonds applied to the body, then the two systems of forces are equivalent from static point of view.

In the case of a mixed situation, if the rigid body is submitted to bonds without being immobilized, the equivalent systems of forces will determine the same reactions, acting successively, in the bonds applied and the same modification of the state of movement of the body.

The transformation of a system of forces into an equivalent system has as objective simplification of the system, that is, its replacement with a simpler system. This operation is called reduction of the system of forces.

3. REDUCTION OF SLIDING VECTORS

One of the basic operations of classical mechanics is the substitution of the system of forces applied to a solid rigid body with the simplest system that produces the same results (effects) in respect of mechanical movement, that is, the simplest equivalent system. The operation by which a system of forces (sliding vectors) is replaced by the simplest equivalent system is called reduction. Reduction of a system of forces applied to a solid rigid body, considered as system of sliding vectors, means its replacement with the simplest possible system that produces the same effect from mechanical point of view on the body.

There are three types of simple vector systems: system made of a single sliding vector (single vector); system made of two parallel sliding vectors, equal in module, and of opposite senses (couple of vectors); system formed of two sliding vectors with con-concurrent and non-parallel supports in space (torsor per se).

Beside the three types of sliding vector systems in classic mechanics, two particular cases of sliding vector systems are studied as well, coplanar vector systems and parallel vector systems that can be coplanar or non-coplanar, respectively. These particular systems of sliding vectors arise in the static and dynamic study of solid rigid bodies, and in the case of Resistance of Materials discipline as well.

In the case of any system of sliding vectors related to a tri-orthogonal benchmark, the problem of establishing the simple system with which it is equivalent arises; that is, which leads to the same result from mechanical point of view on a solid rigid body. For this, the sliding vector system characteristics are established by calculation, in relation to the considered benchmark.

In a tri-orthogonal benchmark, the resultant, and the resultant moment are called characteristics of a system, in relation with the origin and scalar product between the resultant and the resultant moment (the second scalar invariant or the scalar of the torsor):

$$\overline{R};\ \ \overline{M}_O;\ \ \overline{R}\cdot\overline{M}_O \tag{1}$$

In the case of any system of sliding vectors related to a tri-orthogonal benchmark, the reduction operation, the replacement of the system with the simplest equivalent system, effectively means calculation of the torsor in any pole, in particular pole *O*, the origin of the benchmark to which the system is related:

$$\tau_O=\left\{\overline{R},\ \ \overline{M}_O\right\} \tag{2}$$

Out of the characteristic of simple systems of sliding vectors, the paper further presents only the characteristics of the system made up of two sliding vectors with non-concurrent and non-parallel supports in space (torsor per se), related to a tri-orthogonal benchmark.

We consider the system of vectors, torsor per se, $\overline{F}_1$ and $\overline{F}_2$ with (Δ_1) and (Δ_2) support lines, non-concurrent and non-parallel in space (any in space):

$$S=\left\{\overline{F}_1,\ \ \overline{F}_2\right\},\ \ (\Delta_1)\cap(\Delta_2)=\varnothing,\ \ \overline{u}_1\cdot\overline{u}_2\neq\pm1 \tag{3}$$

where (Δ_1) and (Δ_2) lines versors were noted $\overline{u}_1$ and $\overline{u}_2$

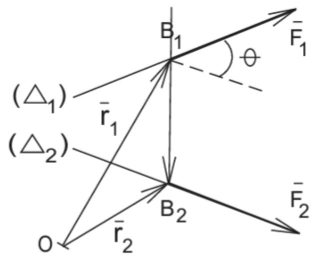

Fig. 1. Torsor per se

$$\overline{R}=\overline{F}_1+\overline{F}_2\neq0 \tag{4}$$

$$\overline{M}_O=\overline{M}_O\left(\overline{F}_1\right)+\overline{M}_O\left(\overline{F}_2\right)=\overline{r}_1\times\overline{F}_1+\overline{r}_2\times\overline{F}_2\neq0 \tag{5}$$

where $\overline{r}_1=\overline{OB_1},\ \ B_1\in(\Delta_1)$ and $\overline{r}_2=\overline{OB_2},\ \ B_2\in(\Delta_2)$

From (4) and (5) for the scalar torsor we obtain:

$$\overline{R}\cdot\overline{M}_O=\left(\overline{F}_1+\overline{F}_2\right)\cdot\left(\overline{r}_1\times\overline{F}_1+\overline{r}_2\times\overline{F}_2\right) \tag{6}$$

$$\begin{aligned}\overline{R}\cdot\overline{M}_O &= \overline{F}_1\left(\overline{r}_1\times\overline{F}_1\right)+\overline{F}_1\left(\overline{r}_2\times\overline{F}_2\right)+\overline{F}_2\left(\overline{r}_1\times\overline{F}_1\right)+\overline{F}_2\left(\overline{r}_2\times\overline{F}_2\right)=\\ &=\overline{F}_1\left(\overline{r}_2\times\overline{F}_2\right)+\overline{F}_2\left(\overline{r}_1\times\overline{F}_1\right)\end{aligned} \tag{7}$$

since the first and last mixed product in (7) are equal with zero, having two identical factors each.

According to the mixed product properties, we further have:

$$\overline{R}\cdot\overline{M}_O=\overline{F}_1\left(\overline{r}_2\times\overline{F}_2\right)-\overline{F}_1\left(\overline{r}_1\times\overline{F}_2\right)=\overline{F}_1\left[\left(\overline{r}_2-\overline{r}_1\right)\times\overline{F}_2\right]=\overline{F}_1\cdot\left(\overline{B_1B_2}\times\overline{F}_2\right)\neq 0 \tag{8}$$

since the three vectors are non-coplanar and non-parallel, two by two.

The resultant of a torsor per se is a free vector, not having a specified support line.

In conclusion, based on equations (4), (5) and (8) the following result can be enounced regarding the characteristics of the simple system of sliding vectors related to a tri-orthogonal benchmark, or a system of torsor per se, $\overline{R}\neq 0;\ \ \overline{M}_O\neq 0;\ \ \overline{R}\cdot\overline{M}_O\neq 0$.

4. ASPECTS IN CASE OF REDUCTION OF THE TORSOR PER SE

The paper further approaches the problem of the central axis of a system of sliding vectors that is reduce to a torsor per se.

The equation:

$$\left(\Delta_C\right):\overline{r}=\frac{\overline{R}\times\overline{M}_O}{R^2}+\lambda\overline{R},\quad \lambda\in\mathbb{R} \tag{9}$$

represents the vectorial equation of a line parallel with the direction of the resultant.

The central axis of a sliding vector system is the locus of points *C* in the range of definition of vectors in relation to which the calculated resultant moment is parallel with the resultant vector, this moment having minimum value.

It is known (according to the second theorem of equivalency), that any system of sliding vectors can be reduced, by elementary operations of equivalency, to one of the simple systems of sliding vectors (torsor per se, single vector or couple of vectors) or, in a particular case, can be balanced (equivalent of zero).

Two systems of sliding vectors, which act successively on a solid rigid body, are equivalent among themselves, if one can be obtained from the other by a succession of elementary operations of equivalency. The term elementary shows that these operations cannot be decomposed in other simple ones.

Thus, according to the second theorem of equivalency, any system of sliding vectors is equivalent to a system of two vectors having as supports any line in space.

Due to this fact, we shall consider bellow such a system, made up of vectors $\overline{F}_1$ and $\overline{F}_2$, the non-parallel supports of which $\left(\Delta_1\right)$ and $\left(\Delta_2\right)$ go through points B_1

and B_2 (Fig. 1).

$$S=\{\overline{F}_1,\overline{F}_2\}, B_1\in(\Delta_1),\ \ B_2\in(\Delta_2),$$

The analytical expressions of vectors $\overline{F}_1$ and $\overline{F}_2$, and the coordinates of points B_1 and B_2 in any benchmark are known.

Relating the system $S=\{\overline{F}_1,\overline{F}_2\}$ to a pole O (in particular being the origin of the benchmark), the elements of the reduction torsor are resultant $\overline{R}$, and the resultant moment $\overline{M}_O$.

$$\overline{R}=\overline{F}_1+\overline{F}_2,\ \ \overline{M}_O=\overline{M}_O\left(\overline{F}_1\right)+\overline{M}_O\left(\overline{F}_2\right),\ \ \overline{R}\cdot\overline{M}_O\neq 0$$

The problem still remains of determining the common perpendicular of the support lines (Δ_1) and (Δ_2).

For this we note with $A_1\in(\Delta_1)$ and $A_2\in(\Delta_2)$ the points that comply with the conditions

$$\overline{A_1A_2}\perp(\Delta_1)\ \text{and}\ \overline{A_1A_2}\perp(\Delta_2)$$

And we note with (Δ) the common perpendicular of the support lines, that is, the line determined by points A_1 and A_2 (Fig. 1).

Relating the vectors to origin *O*, the following vectorial equations can be written:

$$\left.\begin{array}{l}\overline{r}_{A_1}=\overline{r}_{B_1}+\overline{B_1A_1}\\ \overline{B_1A_1}=\lambda_1\overline{F}_1\end{array}\right\}\Rightarrow\overline{r}_{A_1}=\overline{r}_{B_1}+\lambda_1\overline{F}_1$$
$$\left.\begin{array}{l}\overline{r}_{A_2}=\overline{r}_{B_2}+\overline{B_2A_2}\\ \overline{B_2A_2}=\lambda_2\overline{F}_2\end{array}\right\}\Rightarrow\overline{r}_{A_2}=\overline{r}_{B_2}+\lambda_2\overline{F}_2\quad,\ \ \lambda_1,\lambda_2\in R,\tag{10}$$

here $\overline{r}$ represents the position vector of the point, indicated by index. Determination of points A_1 and A_2 is reduced to finding the scalars λ_1,λ_2.

Since we considered that $A_1\ A_2$ is the common perpendicular of the support lines of vectors $\overline{F}_1$ and $\overline{F}_2$, we can write:

$$\overline{A_1A_2}\cdot\overline{F}_1=0,\ \ \overline{A_1A_2}\cdot\overline{F}_2=0$$

or

$$\left(\overline{r}_{A_2}-\overline{r}_{A_1}\right)\cdot\overline{F}_1=0,\ \left(\overline{r}_{A_2}-\overline{r}_{A_1}\right)\cdot\overline{F}_2=0,\tag{11}$$

Subtracting from equations 10), the first equation from the second, we obtain:

$$\overline{r}_{A_2}-\overline{r}_{A_1}=\overline{r}_{B_2}-\overline{r}_{B_1}+\lambda_2\overline{F}_2-\lambda_1\overline{F}_1,\tag{12}$$

From (2) and (3) results:

$$\begin{cases}\left(\overline{r}_{B_2}-\overline{r}_{B_1}+\lambda_2\overline{F}_2-\lambda_1\overline{F}_1\right)\cdot\overline{F}_1=0\\ \left(\overline{r}_{B_2}-\overline{r}_{B_1}+\lambda_2\overline{F}_2-\lambda_1\overline{F}_1\right)\cdot\overline{F}_2=0\end{cases},$$

or, substituting the difference $\overline{r}_{B_2}-\overline{r}_{B_1}$ cu $\overline{B_1B_2}$:

$$\begin{cases}\overline{B_1B_2}\cdot\overline{F}_1+\lambda_2\overline{F}_1\cdot\overline{F}_2-\lambda_1\overline{F}_1^2=0\\ \overline{B_1B_2}\cdot\overline{F}_2+\lambda_2\overline{F}_2^2-\lambda_1\overline{F}_1\cdot\overline{F}_2=0\end{cases},$$

This being an algebraic system with unknown λ_1,λ_2.

The determinant of the system is calculated:

$$\Delta=\overline{F}_1^2\overline{F}_2^2-\left(\overline{F}_1\cdot\overline{F}_2\right)^2=\left|\overline{F}_1\times\overline{F}_2\right|^2\neq 0,$$

since vectors $\overline{F}_1$ are non-parallel:

$$\begin{aligned}\Delta_1&=-\left(\overline{B_1B_2}\cdot\overline{F}_1\right)\overline{F}_2^2+\left(\overline{B_1B_2}\cdot\overline{F}_2\right)\left(\overline{F}_1\cdot\overline{F}_2\right)\\ \Delta_2&=\overline{F}_2^2\left(\overline{B_1B_2}\cdot\overline{F}_2\right)-\left(\overline{F}_1\cdot\overline{F}_2\right)\left(\overline{B_1B_2}\cdot\overline{F}_1\right)\end{aligned},$$

It results:

$$\lambda_1=\frac{\Delta_1}{\Delta}=\frac{\left(\overline{F}_1\cdot\overline{F}_2\right)\left(\overline{B_1B_2}\cdot\overline{F}_2\right)-\overline{F}_2^2\left(\overline{B_1B_2}\cdot\overline{F}_1\right)}{\left|\overline{F}_1\times\overline{F}_2\right|^2},\tag{13,a}$$

$$\lambda_2=\frac{\Delta_2}{\Delta}=\frac{\overline{F}_1^2\left(\overline{B_1B_2}\cdot\overline{F}_2\right)-\left(\overline{F}_1\cdot\overline{F}_2\right)\left(\overline{B_1B_2}\cdot\overline{F}_1\right)}{\left|\overline{F}_1\times\overline{F}_2\right|^2},\tag{13,b}$$

by substituting λ_1 and λ_2 in (10) we obtain the position vectors of points A_1 and A_2.

Relative to pole *O*, the equation of the central axis of the system $S=\left\{\overline{F}_1,\overline{F}_2\right\}$ is:

$$\left(\Delta_C\right):\overline{r}=\frac{\overline{R}\times\overline{M}_O}{\left(\overline{R}\right)^2}+\lambda\overline{R},\quad \lambda\in\mathbb{R}$$

Further, we shall preferably determine equation of the central axis with the help of the torsor calculated in relation to the pole (point) A_1, $\tau_{A_1}=\left\{\overline{R},\ \ \overline{M}_{A_1}\right\}$.

$$\left(\Delta_C\right):\overline{r}=\frac{\overline{R}\times\overline{M}_{A_1}}{\left(\overline{R}\right)^2}+\lambda\overline{R},\quad \lambda\in\mathbb{R}\,, \tag{14}$$

where:

$$\overline{R}=\overline{F}_1+\overline{F}_2,\quad \overline{M}_{A_1}=\overline{M}_{A_1}\left(\overline{F}_1\right)+\overline{M}_{A_1}\left(\overline{F}_2\right)=\overline{M}_{A_1}\left(\overline{F}_2\right),\quad \overline{R}\cdot\overline{M}_{A_1}\neq 0$$

we have:

$$\begin{aligned}
\overline{R}\times\overline{M}_{A_1}&=\left(\overline{F}_1+\overline{F}_2\right)\times\overline{M}_{A_1}\left(\overline{F}_2\right)=\left(\overline{F}_1+\overline{F}_2\right)\times\left(\overline{A_1A_2}\times\overline{F}_2\right)=\\
&=\overline{F}_1\times\left(\overline{A_1A_2}\times\overline{F}_2\right)+\overline{F}_2\times\left(\overline{A_1A_2}\times\overline{F}_2\right)=\\
&=\left(\overline{F}_1\cdot\overline{F}_2\right)\overline{A_1A_2}-\left(\overline{F}_1\cdot\overline{A_1A_2}\right)\overline{F}_2+\left(\overline{F}_2\right)^2\overline{A_1A_2}-\left(\overline{F}_2\cdot\overline{A_1A_2}\right)\overline{F}_2=,\\
&=\left(\overline{F}_1\cdot\overline{F}_2\right)\overline{A_1A_2}+\left(\overline{F}_2\right)^2\overline{A_1A_2}=\left[\overline{F}_1\cdot\overline{F}_2+\left(\overline{F}_2\right)^2\right]\overline{A_1A_2}
\end{aligned}$$

since $\overline{A_1A_2}\cdot\overline{F}_1=0,\ \ \overline{A_1A_2}\cdot\overline{F}_2=0$

Substituting the result obtained in equation (14), we have:

$$\left(\Delta_C\right):\overline{r}=\frac{\left[\overline{F}_1\cdot\overline{F}_2+\left(\overline{F}_2\right)^2\right]\overline{A_1A_2}}{\left(\overline{F}_1+\overline{F}_2\right)^2}+\lambda\left(\overline{F}_1+\overline{F}_2\right),\quad \lambda\in\mathbb{R}\,,$$

$$\left(\Delta_C\right):\overline{r}=p_1\cdot\overline{A_1A_2}+\lambda\left(\overline{F}_1+\overline{F}_2\right),\quad \lambda\in\mathbb{R}\,, \tag{15}$$

where the scalar was noted:

$$p_1=\frac{\left[\overline{F}_1\cdot\overline{F}_2+\left(\overline{F}_2\right)^2\right]}{\left(\overline{F}_1+\overline{F}_2\right)^2},$$

If we determine the equation of the central axis with the help of the reduction torsor in relation to point A_2, $\tau_{A_2} = \left\{ \overline{R},\ \overline{M}_{A_2} \right\}$, following and analogous calculation, we obtain:

$$\left(\Delta_C\right) : \overline{r} = p_2 \cdot \overline{A_2 A_1} + \lambda\left(\overline{F}_1 + \overline{F}_2\right),\ \ \lambda \in \mathbb{R}, \tag{16}$$

where we noted the scalar:

$$p_2 = \frac{\left[\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_1\right)^2\right]}{\left(\overline{F}_1 + \overline{F}_2\right)^2},$$

We notice that we have:

$$\begin{aligned} p_1 + p_2 &= \frac{\left[\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_2\right)^2\right]}{\left(\overline{F}_1 + \overline{F}_2\right)^2} + \frac{\left[\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_1\right)^2\right]}{\left(\overline{F}_1 + \overline{F}_2\right)^2} = \\ &= \frac{\left(\overline{F}_1\right)^2 + 2\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_1\right)^2}{\left(\overline{F}_1 + \overline{F}_2\right)^2} = \frac{\left(\overline{F}_1 + \overline{F}_2\right)^2}{\left(\overline{F}_1 + \overline{F}_2\right)^2} = 1 \end{aligned},$$

Which means that $p_1 \cdot \overline{A_1 A_2}$ and $p_2 \cdot \overline{A_2 A_1}$ are vectors with the peaks in the same point, which we note *C*, point that belongs to the common perpendicular $A_1\ A_2$.

In conclusion, the equation of the central axis in any benchmark with origin *O*:

$$\left(\Delta_C\right) : \overline{r} = \overline{r}_C + \lambda\left(\overline{F}_1 + \overline{F}_2\right),\ \ \lambda \in \mathbb{R}, \tag{17}$$

can be written, where $\overline{r}_C$ is the position vector of point *C*, and:

$$\overline{r}_C = \overline{r}_{A_1} + p_1 \cdot \overline{A_1 A_2}\text{ , conform (15)}$$

or

$$\overline{r}_C = \overline{r}_{A_2} + p_2 \cdot \overline{A_2 A_1}\text{ , conform (16)}$$

If we take in consideration: $p_1 + p_2 = 1$, $\overline{A_1 A_2} = \overline{r}_{A_2} - \overline{r}_{A_1} = -\overline{A_2 A_1}$, for the position vector of point *C* we obtain:

$$\overline{r}_C = p_2 \cdot \overline{r}_{A_1} + p_1 \cdot \overline{r}_{A_2},$$

where p_1 and p_2 are given by equations (15) and (16).

The central axis is a line perpendicular on the common perpendicular (*Δ*) of vectors $\overline{F}_1$ and $\overline{F}_2$, having in common with those point *C*.

We consider the following example.

For the simple system formed of the two sliding vectors with non-concurrent and non-parallel supports in space in Fig. 3, the intersection points of the central axis with the common perpendicular of the two vectors is to be determined.

The side of a cube *a* and the magnitudes of the two vectors $F_1 = F_2 = P\sqrt{2}$ are given.

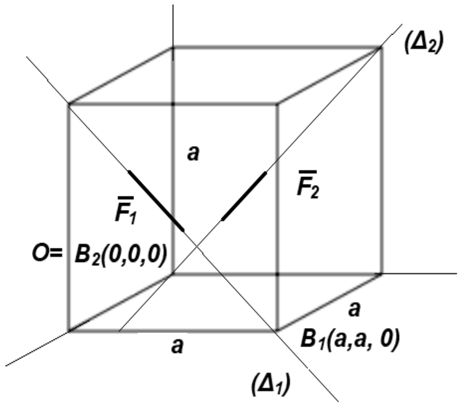

Fig. 3. System of vectors

Thus we have:

$$\overline{F}_1 = -P\overline{j} + P\overline{k}, \quad \overline{F}_2 = P\overline{j} + P\overline{k},$$

The resultant and the resultant moment are:

$$\overline{R} = \overline{F}_1 + \overline{F}_2 = 2P\overline{k}, \quad \overline{M}_0 = \overline{M}_O(\overline{F}_1) + \overline{M}_O(\overline{F}_1) = aP\overline{i} - aP\overline{j} + aP\overline{k}$$

The equation of the central axis is:

$$\frac{M_{Ox} - yR_z + zR_y}{R_x} = \frac{M_{Oy} - zR_x + xR_z}{R_y} = \frac{M_{Oz} - xR_y + yR_x}{R_z}$$

whence:

$$\frac{aP - 2yP}{0} = \frac{aP + x2P}{0} = \frac{-aP}{2P}$$

and the equation of a line parallel with Oz obtained by the intersection of planes $y = a/2$ and $x = a/2$ is obtained.

In order to determine the common point between the common central and perpendicular axis of the two vectors, we shall use the previously determined equations.

λ_1 and λ_2 being determined, the position vectors of the intersection points of the common perpendicular *(Δ)* with the supports *(Δ1)* and *(Δ2)* of the two vectors will be written.

$$\overline{r}_{A_1} = \overline{r}_{B_1} + \lambda_1\overline{F}_1 = a\overline{i} + (a/2)\overline{j} + (a/2)\overline{k}$$
$$\overline{r}_{A_2} = \overline{r}_{B_2} + \lambda_2\overline{F}_2 = (a/2)\overline{j} + (a/2)\overline{k}$$

For the position vector of the intersection point between the central axis and the common perpendicular of the supports of the two vectors, this equation is used:

$$\overline{r}_C = p_2 \cdot \overline{r}_{A_1} + p_1 \cdot \overline{r}_{A_2} ,$$

where p_1 and p_2 are given by the equations (15) and (16).

$$p_1 = \frac{\left[\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_2\right)^2\right]}{\left(\overline{F}_1 + \overline{F}_2\right)^2} = \frac{1}{2} \quad \text{şi} \quad p_2 = \frac{\left[\overline{F}_1 \cdot \overline{F}_2 + \left(\overline{F}_1\right)^2\right]}{\left(\overline{F}_1 + \overline{F}_2\right)^2} = \frac{1}{2}$$

whence:

$$\overline{r}_C = \frac{1}{2}\left(a\overline{i} + \frac{a}{2}\overline{j} + \frac{a}{2}\overline{k} \right) + \frac{1}{2}\left(\frac{a}{2}\overline{j} + \frac{a}{2}\overline{k} \right) = \frac{a}{2}\overline{i} + \frac{a}{2}\overline{j} + \frac{a}{2}\overline{k} ,$$

5. CONCLUSIONS

The paper presents aspects regarding reduction of sliding vectors, namely in the case of a system formed of two sliding vectors with mom-concurrent and non-parallel supports in space (torsor per se), namely the intersection between the central axis is a perpendicular line on the common perpendicular.

The characteristics of the system of sliding vectors have been determined by calculation in relation with the benchmark considered, the resultant, the resultant moment in relation with the origin and scalar product between the resultant and the resultant moment (the second scalar invariant or the scalar of the torsor).

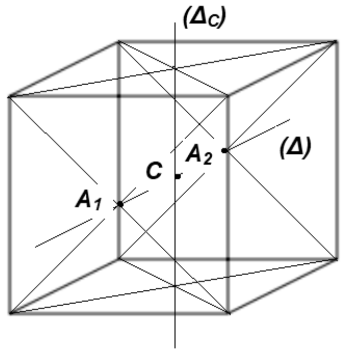

Fig. 4. Intersection between (Δ) and (Δ_C)

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