DETERMINATION OF INERTIA MOMENTS IN AN EVEN BRACED FRAME GIRDER BASED ON REDUCTION TO SYSTEMS OF MATERIAL POINTS WITH THE SAME INERTIAL CHARACTERISTICS AS THOSE OF THE GIRDER BARS

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Abstract: The paper attempts to determine inertia moments in an even braced frame girder based on reduction to systems of material points with the same initial characteristics as those of the girder bars and verification of the results obtained with this method.

Keywords: inertia moments, even braced frame girder.

1. INTRODUCTION

Constructions (statically determined), named braced frame girders, are made up of straight bars articulated in knots and making up plane or spatial structures (frames); they can successfully replace massive structural plates or volumes compared to which they have similar capacities of taking over mechanical loads, but much smaller material consumptions and, far less weights implicitly. In constructions, braced frame girders are structural elements, having the role of taking over loads from other structural or non-structural elements, transmitting those to outriggers.

Inertia is the general characteristic of bodies to to oppose changes in resting state or uniform rectilinear movement. Physical magnitude called mass is associated to inertia properties of a body.

The higher the inertia of a body, the higher is its mass. Inertia moment is a physical magnitude characterizing mass distribution around an axis. Thus, the inertia of bodies is mainly related to matter, but also to the way in which this is distributed in space.

Unlike the mass centre, an inertial invariant, as well as the mass of the body, the magnitudes of inertia moments are modified by changing the system of reference.

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In this paper the calculation method will be shown by a reduction method, by which a body is reduced to a system of material points. In this case, the bar of the braced frame girder is reduced to a system of three material points.

In conclusion, the study of any mechanical system must include a phase in which inertia properties of the bodies making up the system in question should be determined.

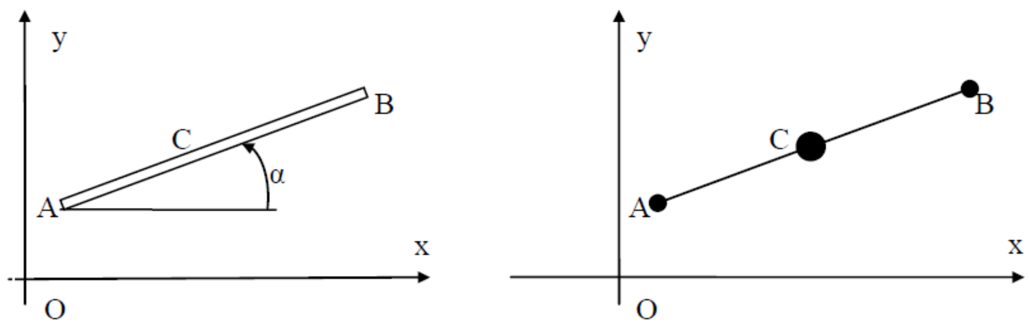
2. DETERMINATION OF INERTIA MOMENTS BASED ON REDUCTION TO SYSTEMS OF MATERIAL POINTS.

The aim of any reduction is to ease the mathematical calculation required to determine the inertia characteristics. All reductions involve bodies with the same mass, the same mass centre and the same inertia matrix as that of the reduced element.

It has been considered that, from the point of view of calculation of the inertia moments and centrifugal moments, bodies can fall into four categories. Of these, bodies of order 1, which can be reduced to a line segment, to a curve segment or a compound curve (a curve has only one dimension), which are also called bars, and the only significant dimension being length, are categories of interest. In order 1 category bodies are all the homogeneous bodies that can be perfectly determined using only one parameter. The straight bar can be considered the only simple known elementary body.

According to the reduction theorem, any straight bar can be reduced to a system made up of three material points, to which the mass of the bar is attributed, thus: the ends of the bar will be attributed one sixth (1/6) of the bar mass, and the rest of two thirds (2/3) will be attributed to the centre of the bar.

For the homogeneous rectilinear bar, situated in plane *xOy*, defined by points $A(x_A, y_A)$ and $B(x_B, y_B)$, Fig. 1, the bar mass being known, , *m*, as well as its length, *l*, and considering that the position of the bar is known, given by the angle it makes with the horizontal direction, *α*, and the coordinates of the ends of the bar, the axial inertia moments are:



a) b)

Fig. 1. Homogeneous straight bar situated in any position in plane xOy
a) as continuous medium; b) as system of material points

- for the model proposed in Fig. 1a:

$$J_x = \frac{m}{3}\left(y_A^2 + y_A y_B + y_B^2\right); J_y = \frac{m}{3}\left(x_A^2 + x_A x_B + x_B^2\right); J_z = J_x + J_y \tag{1}$$

- for the model proposed in Fig. 1b:

$$\begin{aligned} J_x &= \frac{m}{6}y_A^2 + \frac{m}{6}y_B^2 + \frac{2m}{3}\left(\frac{y_A + y_B}{2}\right)^2 = \frac{m}{3}\left(y_A^2 + y_B^2 + y_A y_B\right) \\ J_y &= \frac{m}{6}x_A^2 + \frac{m}{6}x_B^2 + \frac{2m}{3}\left(\frac{x_A + x_B}{2}\right)^2 = \frac{m}{3}\left(x_A^2 + x_B^2 + x_A x_B\right) \\ J_z &= J_x + J_y \end{aligned} \tag{2}$$

It is noticed that the model proposed in Fig. 1b, for the replacement of the bar (reduction of the bar), has all the axial inertia moments equal to those of the given bar.

Furthermore, for the girder in Fig. 2, using the method of reduction of the girder bars to systems of material points (Fig. 3), the moments of axial inertia will be determined. The bars of the girder are homogeneous, all having the same density.

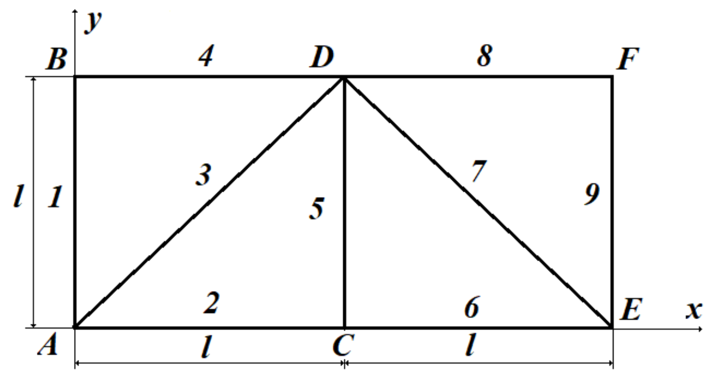


Fig. 2. Braced frame girder

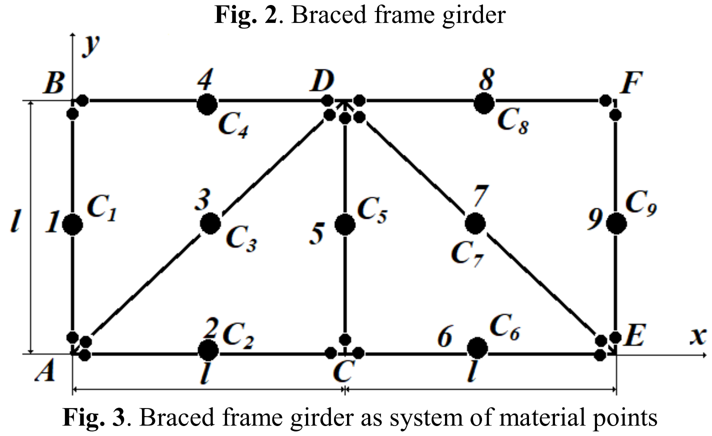


Fig. 3. Braced frame girder as system of material points

The masses of the girder knots, considering the way of attributing the mass to each bar made up of the system of three material points (the ends of the bars will be attributed one sixth (1/6) of the mass of the bar, and the mass centre of the bar the rest of two thirds (2/3)), are:

$$\begin{aligned}
m_A &= m_E = \frac{m}{6}+\frac{m}{6}+\frac{m\sqrt{2}}{6}=\frac{2m}{6}+\frac{m\sqrt{2}}{6}=\frac{m}{3}+\frac{m\sqrt{2}}{6}\\
m_B &= m_F = \frac{m}{6}+\frac{m}{6}=\frac{2m}{6}=\frac{m}{3}\\
m_C &= \frac{m}{6}+\frac{m}{6}+\frac{m}{6}=\frac{3m}{6}=\frac{m}{2}\\
m_D &= \frac{m}{6}+\frac{m}{6}+\frac{m}{6}+\frac{m\sqrt{2}}{6}+\frac{m\sqrt{2}}{6}=\frac{3m}{6}+\frac{2m\sqrt{2}}{6}=\frac{m}{2}+\frac{m\sqrt{2}}{3}\\
m_{C_1} &= m_{C_2}=m_{C_4}=m_{C_5}=m_{C_6}=m_{C_8}=m_{C_9}=\frac{2m}{3}\\
m_{C_3} &= m_{C_7}=\frac{2m\sqrt{2}}{3}
\end{aligned} \tag{3}$$

Axial inertia moments will be:

$$\begin{aligned}
J_x &= \left(3\frac{2m}{3}+2\frac{2m\sqrt{2}}{3}\right)\left(\frac{l}{2}\right)^2+\left(2\frac{m}{3}+2\frac{2m}{3}+\frac{m}{2}+\frac{m\sqrt{2}}{3}\right)l^2=\\
&=2\frac{ml^2}{4}+\frac{4m\sqrt{2}l^2}{3\cdot 4}+\frac{6ml^2}{3}+\frac{ml^2}{2}+\frac{m\sqrt{2}l^2}{3}=\\
&=\frac{ml^2}{2}+2ml^2+\frac{ml^2}{2}+\frac{2m\sqrt{2}l^2}{3}=3ml^2+\frac{2m\sqrt{2}l^2}{3}=\frac{ml^2}{3}\left(9+2\sqrt{2}\right)
\end{aligned} \tag{4}$$

$$\begin{aligned}
J_y &= \left(\frac{2m}{3}+\frac{2m}{3}+\frac{2m\sqrt{2}}{3}\right)\left(\frac{l}{2}\right)^2+\left(\frac{m}{2}+\frac{m\sqrt{2}}{3}+\frac{2m}{3}+\frac{m}{2}\right)l^2+\\
&+\left(\frac{2m}{3}+\frac{2m}{3}+\frac{2m\sqrt{2}}{3}\right)\left(\frac{3l}{2}\right)^2+\left(\frac{m}{3}+\frac{2m}{3}+\frac{m}{3}+\frac{m\sqrt{2}}{6}\right)(2l)^2=\\
&=\frac{ml^2}{3}\left(31+8\sqrt{2}\right)
\end{aligned} \tag{5}$$

$$J_z = J_x+J_y=\frac{ml^2}{3}\left[\left(9+2\sqrt{2}\right)+\left(31+8\sqrt{2}\right)\right]=\frac{ml^2}{3}\left(40+10\sqrt{2}\right) \tag{6}$$

3. VERIFICATION OF TH RESULTS OBTAINED BASED ON REDUCTION TO SYSTEMS OF MATERIAL POINTS

The results obtained based on reduction to systems of material points will be verified with the help of the equations (1). Thus we obtain:

$$J_x^1 = \frac{m}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{ml^2}{3}$$

$$J_x^2 = \frac{m}{3}\left(0^2 + 0 \cdot 0 + 0^2\right) = 0$$

$$J_x^3 = \frac{m\sqrt{2}}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{m\sqrt{2}l^2}{3}$$

$$J_x^4 = \frac{m}{3}\left(l^2 + l \cdot l + l^2\right) = \frac{m3l^2}{3} = ml^2$$

$$J_x^5 = \frac{m}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{ml^2}{3}$$

$$J_x^6 = \frac{m}{3}\left(0^2 + 0 \cdot 0 + 0^2\right) = 0$$

$$J_x^7 = \frac{m\sqrt{2}}{3}\left(l^2 + l \cdot 0 + 0^2\right) = \frac{m\sqrt{2}l^2}{3}$$

$$J_x^8 = \frac{m}{3}\left(l^2 + l \cdot l + l^2\right) = \frac{m3l^2}{3} = ml^2$$

$$J_x^9 = \frac{m}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{ml^2}{3}$$

Whence:

$$J_x = \sum_{i=1}^{9} J_x^i = 3ml^2 + \frac{2m\sqrt{2}l^2}{3} = \frac{ml^2}{3}\left(9 + 2\sqrt{2}\right) \qquad (7)$$

$$J_y^1 = \frac{m}{3}\left(0^2 + 0 \cdot 0 + 0^2\right) = 0$$

$$J_y^2 = \frac{m}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{ml^2}{3}$$

$$J_y^3 = \frac{m\sqrt{2}}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{m\sqrt{2}l^2}{3}$$

$$J_y^4 = \frac{m}{3}\left(0^2 + 0 \cdot l + l^2\right) = \frac{ml^2}{3}$$

$$J_y^5 = \frac{m}{3}\left(l^2 + l \cdot l + l^2\right) = \frac{3ml^2}{3}$$

$$J_y^6 = \frac{m}{3}\left(l^2 + l \cdot 2l + 4l^2\right) = \frac{7ml}{3}$$

$$J_y^7 = \frac{m\sqrt{2}}{3}\left(l^2 + l \cdot 2l + 4l^2\right) = \frac{7m\sqrt{2}l^2}{3}$$

$$J_y^8 = \frac{m}{3}\left(l^2 + l \cdot 2l + 4l^2\right) = \frac{7ml^2}{3}$$

$$J_y^9 = \frac{m}{3}4l\left(0^2 + 2l \cdot 2l + 4l^2\right) = \frac{12ml^2}{3}$$

Whence:

$$J_y = \sum_{i=1}^{9} J_y^i = \frac{31ml^2}{3} + \frac{8m\sqrt{2}l^2}{3} = \frac{ml^2}{3}\left(31 + 8\sqrt{2}\right) \quad (8)$$

$$J_z = J_x + J_y = \frac{ml^2}{3}\left[\left(9 + 2\sqrt{2}\right) + \left(31 + 8\sqrt{2}\right)\right] = \frac{ml^2}{3}\left(40 + 10\sqrt{2}\right) \quad (9)$$

4. CONCLUSIONS

The operation of reduction of bodies refers to replacing a body, which has a certain degree of complexity, to a less complex body, but with the same specific properties as the initial body, more exactly, with the same inertia characteristics.

The aim of any reduction is to ease the mathematical calculation required to determine the inertia properties. The method that can easily be implemented in automatic calculation programs represents a valuable tool in design.

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