NUMERICAL INVESTIGATION OF OPTIMIZED STIFFENED PLATES WITH DAMAGED STIFFENERS

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Abstract: This paper deals with the buckling stability of stiffened plates under longitudinal compression with simply supported conditions within the framework of plate theory. The main objective of the finite element analysis is to investigate some behaviors of these thin-wall structures. The structure of stiffened plate is widespread, from which the version used in this paper has already been optimized for uniaxial compression, some design variables and the cost of welding, and the objective function to be minimized is defined as the material cost. The effect of stiffener damage caused by corrosion can be investigated in FE models of the optimized stiffed plate structure. The buckling shape modes for damaged structures can be compared with the damaged-free ones so that the changes in load capacity can be predicted.

Keywords: optimized stiffened plates; FEA, buckling mode shapes, damaged stiffeners

1. INTRODUCTION

Stiffened plates have many applications in civil, aerospace, automobile and marine structures. These plates subjected to uniaxial or biaxial compression, in-plane bending and shear, lateral pressure, hydrostatic pressure, concentrated or distributed on a line, uniformly distributed static, dynamic loads and used in high temperature many times over their lifetime need to be strengthened to increase the load-carrying capacity of plates by using stiffeners. In the case of moving machines, dynamic analysis is essential when examining the structural elements of the machine [4, 5].

Buckling can also occur even though the applied loads in structures are well below those needed to cause failure in them. In order to increase the strength and avoid any buckling of stiffened plates, buckling analysis of thin-wall structures is used to predict various modes of buckling and derived the critical buckling load. The out of plane buckling deformations of the base plate reach a critical level when the structure

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is subjected to critical loads and the further loading may cause significant deformations [2]. The effect of initial imperfections and residual welding stresses are considered in design methods are predicted loads likely to act on such type of structure.

Our design constraints of the investigated stiffened plate structure are the follows [6]:

- Global buckling of the stiffened plate. The effect of initial imperfections and residual welding stresses are considered by defining buckling curves for a reduced slenderness.
- Single panel buckling. This constraint eliminates the local buckling of the base plate parts between the stiffeners.
- Local and torsional buckling of stiffeners. These instability phenomena depend on the shape of stiffeners. The actual torsional buckling stress can be calculated in the function of the reduced slenderness.
- Distortion constraint. Large deflections due to weld shrinkage should be avoided.

In general, an efficient technique is the finite element analysis (FEA) for buckling analysis of stiffened plates to investigate the effect of initial geometric imperfection on the load displacement response and so on. Furthermore, FEA can be applied to the study of the effects of corrosion or other damage in stiffened plate structures focusing on elastic buckling strength [8]. In addition, leaving one of the ribs out of the FE model the influence of damage can also be investigated in simple models. In such cases, the simplification is allowed if the corrosion concentrates along the base of a rib, as that becomes a non-load-bearing element and provided that the basement plate is intact. With this approach, there are significant differences among the results from modified FE models of the stiffened plate which are also easily comparable to examine the change in load capacity. As the corrosion effects also change the natural frequencies of the structure, a linear perturbation analysis for the damaged stiffened plate [3], [6] would need to be performed again and new FE models created. Therefore, it is important that the structures are properly designed and constructed taking into account corrosion damage during their lifespan.

2. STRUCTURAL PARAMETERS OF STIFFENED PLATE MODEL

Our investigation is made on the buckling stability of the thin-wall structure made up of a base plate and some stiffeners with simple support (SSSS) conditions on all the edges of the base plate, subjected to uniaxial compression on two edges of the plate through numerical simulation. As shown in Fig. 1, the lengths of the plate along the x-axis and y-axis are denoted by L and B, respectively. Besides, the plate possesses a uniformly t_f thickness along the z-axis. The displacement boundary conditions on the edges at $x = 0$ and $x = L$ are given in the form.

$$w = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0 \tag{1}$$

and the boundary conditions on the edges at $y = 0$ and $y = B$ are prescribed according to

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0, \tag{2}$$

where $w = w(x, y)$ is the displacement in the z direction.

Fig. 1 shows a stiffened plate loaded by uniaxial compression. The magnitude of the compression force is $\mathrm{N}=1.2\times10^{7}\ \mathrm{N}$. Thus, a uniformly distributed static load can also be applied to FEA boundary conditions.

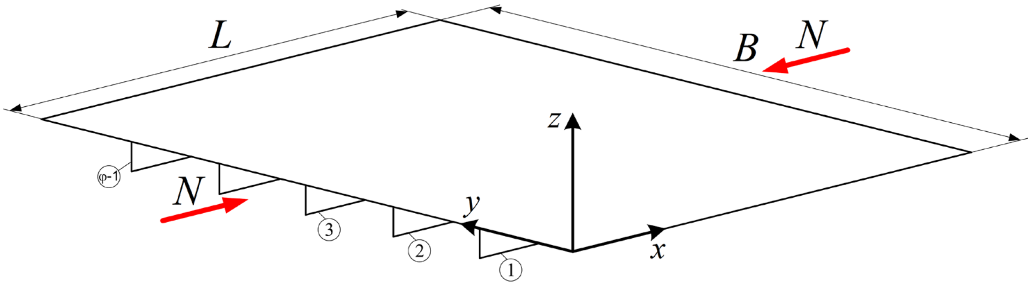

Fig. 1. Longitudinally stiffened plate loaded by uniaxial compression

The given geometrical data of the base plate are width $B = 6000\,mm$ and length $L = 4000\,mm$. The stiffened plate structure is made of steel, having Young's modulus of E = 2.1×10^{5} MPa, Poisson's ratio of 0.3, density of $\rho = 7.85\times10^{-9}\ \mathrm{t/mm^3}$. The yield stress is $\sigma_y = 235\,MPa$.

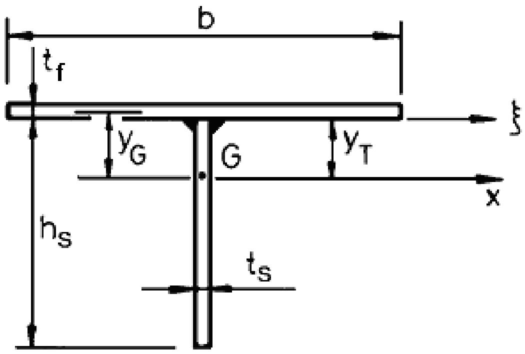

Fig. 2. Dimensions of a flat stiffener

The optimum results for different fabrication costs calculated by Excel Solver NLP which uses a gradient method where the unknowns – t_f (the thicknesses of the base plate), and t_s the stiffener and φ-1 (the number of the ribs) – are limited in size and the welding technology is SAW – see details in Fig. 2 and paper [6].

Table 1. **Optimum results for the plate with flat stiffeners**

Number of the stiffeners (φ-1)	t_f [mm]	t_s [mm]	h_s [mm]	φ
5	15	13	182	6
9	11	12	168	10
28	5	10	140	29

The higher production cost gives a thicker base plate with fewer ribs. An optimum result for the material cost is shown in Table 1 thus, the dimensions and the layout of the ribs are given for analyses [6].

3. ANALYSIS RESULTS OF THE STIFFENED PLATES

The main concept is the subdivision of the model of structure into non-overlapping components of simple shaped geometry called finite elements with well-defined stress displacement relationships. In the numerical simulation, the plates with stiffeners are divided into finite elements and a conventional shell model, an 8-node shell element (S8R) is employed for buckling investigation in the finite element analysis (FEA). More detailed descriptions of finite element procedures can be found in Bathe's book [1].

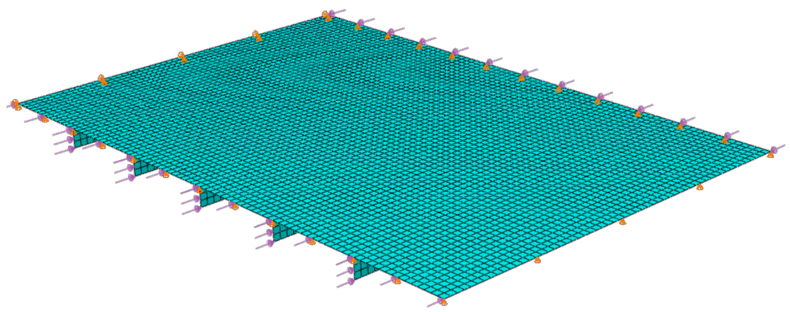

Fig. 3. Abaqus model of a stiffened plate with boundary conditions

It is also noted that the base plate is subjected to a uniformly distributed static load $f = 1\ N/mm$ along the edges paralleled to the y-axis (on the edges at $x = 0$ and $x = L$) and uniformly distributed static loads which are properly converted based on Table 1 are subjected on the corresponding edges of the stiffeners so that the loading is applied as a "shell edge load" for buckling mode shapes in the commercial program Abaqus [8] as shown in Fig 3.

In general, the eigenvalue-eigenvector problem for the n-DOF undamped system is given follows:

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{q}=\mathbf{0}, \tag{3}$$

where $\mathbf{K}$ denotes the stiffness matrix, $\mathbf{M}$ is the mass matrix. The natural frequency ω and the eigenvector $\mathbf{q}$ are the unknowns. The buckling mode shapes are determined in a similar way by solving a linear equation system in the software Abaqus [8]. In structural engineering, the basic form of the linear buckling analysis is given by

$$\mathbf{K}\mathbf{\Phi}_i=\lambda_i\mathbf{S}\mathbf{\Phi}_i \qquad (i=1,2,\ldots,n), \tag{4}$$

where $\mathbf{\Phi}_i$ is the i$^{\text{th}}$ mode shape and so the i$^{\text{th}}$ column of the eigenvector's matrix, λ_i is the eigenvalue belonging to eigenvector $\mathbf{\Phi}_i$ and $\mathbf{S}$ is the stress-stiffness matrix. To investigate the eigenvalue buckling prediction, a Lanczos iteration method is performed to extract eigenvalues – for details, see the book [1]. For the stiffened plate structure with SSSS support conditions on four edges and 5 stiffeners, the buckling mode shape according to the first eigenvalue ($\lambda_1=2762.8$) is shown in Fig. 4. In this case, the maximum load (the critical buckling load) can be estimated to prevent structural instability or collapse. For simplicity, the estimated critical buckling load is given from the first eigenvalue 2762.8 N/mm multiplied by a scale factor of 1. In most cases, we need to perform a number of analyses and use the first few buckling mode shapes to investigate the sensitivity of structures to imperfections. That means that the ideal geometry is perturbed by scales displacement fields deriving from the buckling mode shapes.

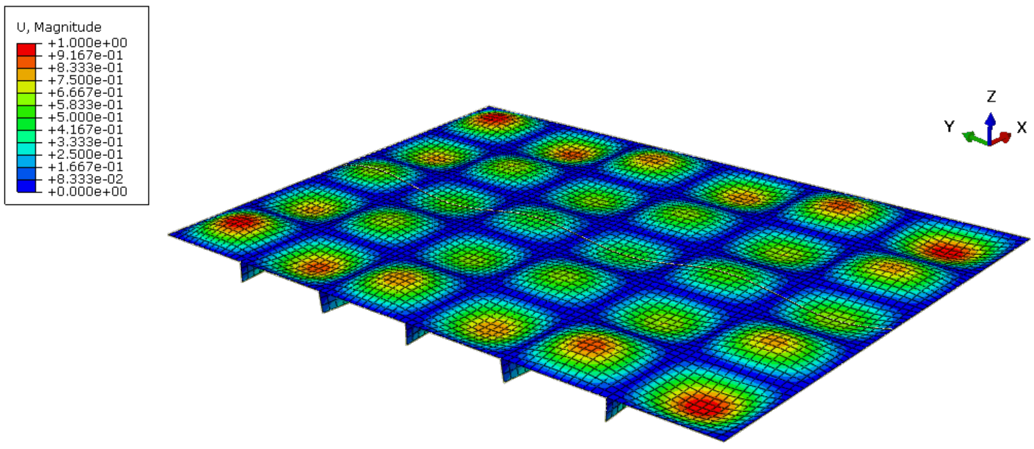

Fig. 4. The first buckling mode shape with SSSS support condition ($\lambda_1=2762.8$)

For the shell edge load magnitude which is calculated from the compression force $N=1.2\times10^{7}\ N$, the effective stresses and displacement magnitude in direction z for stiffened plate model with the imperfection magnitude is taken as 0.5%, 1.0% and 1.5% of the base plate thickness are easy to compute. The maximum values that occurred in the structures are listed in Table 2. The Abaqus is capable to perform a load-displacement analysis and to investigate the effect of initial geometric imperfection on the load-displacement response using the Riks algorithm [8]. The strongly nonlinear stability problems and post-buckling problems, both with stable and unstable behavior can be also solved by using this method.

Table 2. **The maximum values of displacements and effective stresses**

φ-1	imperfection	u_{max} [mm]	σ_{max} [MPa]
5	none	0.000	117.8
	0.5%	0.048	118.7
	1%	0.096	119.5
	1.5%	0.144	120.4
9	none	0.000	142.6
	0.5%	0.031	143.2
	1%	0.063	143.8
	1.5%	0.094	144.4
28	none	0.000	173.4
	0.5%	0.002	173.6
	1%	0.003	173.7
	1.5%	0.005	173.9

Provided that some ribs are considered non-load-bearing elements due to damage, they are omitted from FE models. The simplification of damages is only allowed if the damage is concentrated along the entire length of a rib, provided that the plate remains intact. The influence of damage can be easily modeled by simply omitting some numbered ribs so that some typical cases can be compared in the following three tables in terms of buckling eigenvalues. In general, the buckling eigenvalues are used to estimate the critical load. Fig. 1 shows the numbering of the stiffeners and the first column of Table 3 denotes the abandoned ribs in the FE model. It is clear from the tables that the load-bearing capacity of the stiffened plate is significantly reduced if one of its ribs is completely damaged. In such cases the critical load is also expected to be halved, making the plate unstable at $N=1.2\times10^{7}\ N$.

Table 3. **The eigenvalues for the 5 ribbed arrangement in the case of damaged stiffeners**

No.	λ_1	λ_2	λ_3	λ_4	λ_5
	2762.8	2813.9	2825.1	2852.4	3003.8
1	819.42	1076.6	1380.5	2313.8	2741.3
2	901.23	1142.0	1423.7	2343.1	2746.2
3	903.83	1144.2	1424.4	2343.3	2747.5
1-2	370.13	535.40	1157.1	1231.6	1688.9
2-3	437.07	558.23	1169.3	1315.1	1735.0
1-2-3-4-5	83.330	308.10	396.45	515.73	749.62

The first five eigenvalues belonging to the buckling mode shapes can be seen in Table 3 in which the numbers of missing stiffeners are denoted in column 1.

Table 4. **The eigenvalues for the 9 ribbed arrangement in the case of damaged stiffeners**

No.	λ_1	λ_2	λ_3	λ_4	λ_5
	2752.1	2869.9	3010.9	3027.8	3160.5
1	884.92	909.40	1218.9	1692.0	1848.5
2	1015.6	1095.7	1289.2	1657.5	1744.2
3	1018.7	1105.2	1290.3	1691.8	1744.7
1-2	387.83	589.29	653.65	952.07	1363.4
2-3	446.98	618.41	730.89	971.30	1463.1
1-2-…-8-9	32.885	121.59	156.39	203.51	295.89

Significant changes are observed in load capacity due to damaged stiffeners, however, note that the model is overestimated the effect of damage. There is possible to develop the FE model, to include other forms of corrosion [9].

Table 5. **The eigenvalues for the 28 ribbed arrangement in the case of damaged stiffeners**

No.	λ_1	λ_2	λ_3	λ_4	λ_5
	2739.2	2830.2	3064.8	3449.4	3889.3
1	1486.8	1508.5	1586.0	1603.5	1756.9
2	1868.5	1905.6	1965.9	2005.4	2167.8
3	1872.1	1907.0	1970.6	2006.3	2186.9
1-2	652.00	663.30	740.83	781.26	865.91
2-3	809.32	849.38	869.69	951.01	1098.9
1-2-…-28	6.1876	22.891	29.452	38.337	55.765

The eigenvalues for the two other arrangements can be seen in Tables 4 and 5 where the larger numbers of stiffeners reduce the decrease in eigenvalues for damaged plates.

4. CONCLUSIONS

This paper is intended to present an implementation of the buckling analysis for an optimized stiffened plate to investigate the influence of damage over simple models. The finite element analysis is a powerful technique that is enabled to compare the different optimized plates with flat stiffeners. 3D models of the plates are considered using the 8-noded doubly curved thick S8R shell elements where the ideal geometry can be perturbed by scales displacement fields deriving from the buckling mode shapes. The eigenvalues show a significant decrease if any of the stiffeners are damaged so that the extent of the reduction in load capacity compared to the damaged-free plates can be seen. Therefore, it is important that the structure is properly designed and provided with an adequate corrosion protection.

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