INCREASING MEASUREMENTS ACCURACY IN MECHANICAL EXPERIMENTS BY USING MATHCAD DATA INTERPOLATION

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Abstract: The paper presents a method for mechanical-type experimental data interpolation using the software MathCAD. It is written and explained a program allowing the processing of real experimental data, affected by experimental errors, obtained during mechanics lab and research activities. Some advantages of interpolation with MathCAD are put into evidence: simplicity of program, decrease of experimental errors, and prediction of theoretical dependencies for the determined physical parameters.

Key words: interpolation, MathCAD, experimental data processing, accuracy, mechanical experiments.

1. INTRODUCTION

Experimental data obtained from measurements of different physical parameters are affected by errors, depending on the quality of measuring instruments and devices, experimenter's experience, external factors, etc [1, 2, 3, 4]. Most often, experimental errors are randomly distributed, giving values placed nearly the theoretical (ideal) curve describing the dependence between the determined parameters. Interpolation is the operation of tracing an experimental dependence curve, between experimental points, after the rule given by majority of points [5, 6]. It is usually made by the experimenter, visually, its accuracy depending on the experimenter's experience. It can be done more rigorously with MathCAD, software containing special interpolation functions/commands – taking into account the characteristics of randomly distributed errors [7. 8, 9, 10, 11]. The work presents these functions, by giving a real situation of obtaining experimental data with errors less than 10%, obtained in usual mechanical measurements in high school/university labs, by inexperienced experimenters.

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2. THE MATHCAD PROGRAM

We will give some experimental data regarding a quadratic dependence of a mobile's coordinate, as a function of moving time. Fist, a time row is defined, having values from 0 to 9 (seconds), with step 1 (second). These values can be considered those determined with perfect accuracy (0% relative errors). The character „:=" is obtained by typing „:"; the character „ .." is obtained by typing „;".

$$t := 0,1 .. 9 \tag{1}$$

Then, a function x(t) is defined, giving the values (in meters) determined with perfect accuracy (0% relative errors).

$$x(t) := t^2 + 2t + 3 \tag{2}$$

In fact, we have an uniformly accelerated object, having acceleration a=2 m/s^2, initial velocity v_0=2 m/s, and initial coordinate 3 m.

We will define two vectors, te and xe, containing experimental data with relative errors distributed within 10% (figure 1).

$$te := \begin{bmatrix} 0 \\ 1.1 \\ 2.1 \\ 2.9 \\ 4 \\ 5.1 \\ 6 \\ 6.9 \\ 8 \\ 9 \end{bmatrix} \quad xe := \begin{bmatrix} 3 \\ 5.5 \\ 12 \\ 18 \\ 29 \\ 41 \\ 51 \\ 60 \\ 75 \\ 112 \end{bmatrix}$$

Fig. 1. Input data

Generation of vectors in figure 1 was obtained by typing "ALT+m" and introducing in the dialog box the number of columns 1, and number of rows 10, respectively.

Now fallows the interpolation program Command lines are given, and then explained.

$$xs1 := cspline(te,xe) \tag{3}$$

$$xs2 := pspline(te,xe) \tag{4}$$

$$xs3 := lspline(te,xe) \tag{5}$$

Three variables are defined and characterized by the values of three functions: cspline, pspline and lspline. Each function returns a vector of second derivatives for data vectors te and xe. This vector becomes the first argument of the interpolation function. The resultant spline curve is cubic, parabolic or linear at the endpoints, depending on the function.

$$m := slope(te,xe) \tag{6}$$

$$b := intercept(te,xe) \tag{7}$$

These functions return the slope and intercept of the line that best fits our data in a least square sense.

$$xregress := regress(te,xe,2) \tag{8}$$

Returns a vector which interpolation uses to find the kth order polynomial that best fits the xe and te data values.

$$xloess := loess\ (te,xe,1) \tag{9}$$

Returns a vector which interpolation uses to find a set of second order polynomials that best fit a neighborhood of the xe and te data values. 1 is the size of the neighborhood.

We will chose a second degree function for interpolation, then define a function of parameters u and z, containing as elements a second degree function together with her first, second and third derivates.

$$F(z,u) := \begin{bmatrix} u_0 + u_1 \cdot z + u_2 \cdot z^2 \\ 1 \\ z \\ z^2 \end{bmatrix} \tag{10}$$

W will define a vector with zero initial components.

$$vg := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

The next function is useful when we have a set of measured xe and te values and we want to fit an arbitrary function to our data.

$$P := genfit(te,xe,vg,F) \tag{12}$$

$$P = \begin{bmatrix} 4.173 \\ 1.046 \\ 1.116 \end{bmatrix} \tag{13}$$

We choose a function g(r) to indicate the function best fitting the experimental data (index obtined by typing "["").

$$g(r) := F(r,P)_0 \tag{14}$$

We give values to index i (starting with 0 in MathCAD) and parameter r (that should be near the limits of t).

$$i := 0,1 .. 9 \qquad r := 1,1.1 .. 10 \qquad (15)$$

The graphics of functions are presented in figure 2. One can see the experimental values with squares, the ideal dependence with interrupted line and the interpolation function with line, fitting/superposing very well.
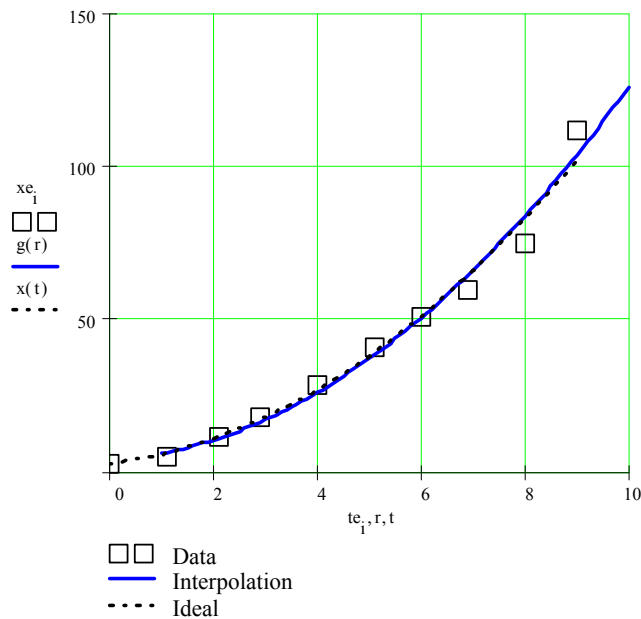

Fig. 2. Graphics of functions

Now we will analyze the same experimental data, but choosing a fourth degree interpolation function. The program will be the same presented before, except the command line in equation 10 (containing a fourth degree polynom) and vector in equation 11 (having 5 elements). The obtained graphics are those in figure 3.

One can see that the two lines are not superposing, interpolation being done too close to the experimental points. This could be good for much smaller experimental errors. Therefore, the experimentator is the one that should appreciate how big the experimental errors are and which type of function could better approximate experimental data.

Coming back to the values of function P, from relation (13), best approximating experimental data, let us remark that interpolation allows the obtaining of the law describing the studied physical phenomenon. However, the values of the second degree polynomial parameters (1.116, 1.046, 4.173) are far away from those describing the ideal dependence (1, 2, 3). A better interpolation/law can be obtained by increasing the measurements accuracy or, more accessible, increasing number of experimental data [12].
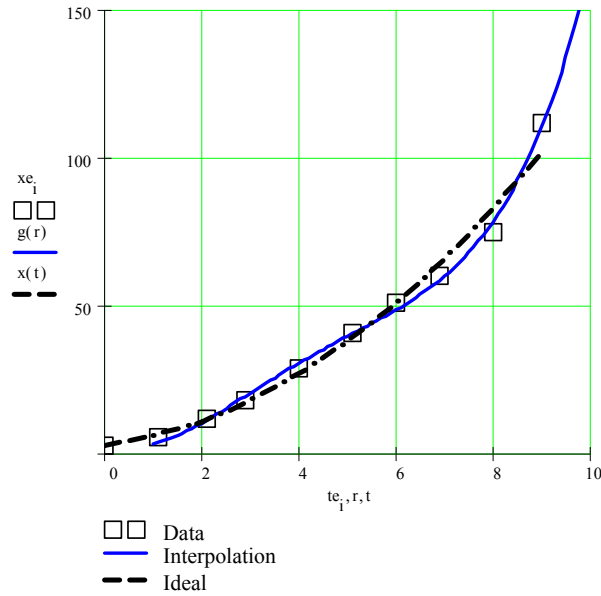
Fig. 3. Data interpolation using a fourth degree function

Figure 4 shows the results obtained for 50 pairs of experimental data. The obtained values of the second degree polynomial parameters (1.048, 1.781, 2.986) are very close to those describing the ideal dependence (relative errors of 4.80, 10.95 and 0.46% respectively), even if the relative experimental errors are still 10%.
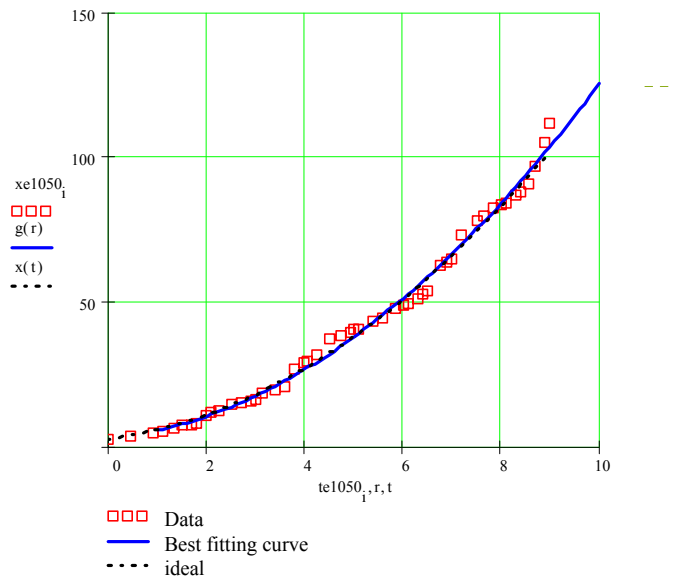

Fig. 4. Graphics of functions for 50 pairs of experimental data

This means that we have finally: a very accurate determination of the initial coordinate, a less accurate determination of the acceleration, and a poorly accurate determination of the initial velocity.

Increasing measurement accuracy can be obtained especially by increasing accuracy (repeat measurements) of values for time and coordinates not fitting curve in figure 4. As an example, repeating measurements for time intervals $(4-5)$ s, $(6-6.5)$ s, $(9-9.5)$ s.

3. CONCLUSIONS

MathCAD is a useful tool for interpolation of experimental data affected by relatively high errors. By interpolation are eliminated both measuring instruments and experimenter caused errors. A more detailed study shows that interpolation can lead to a strong decrease of experimental errors (for example, from 10% to 1%) and to a rigorously prediction of the governing law, by a simple increasing of experimental data number.

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